

If the mass of the Sun is  $M_\odot$  and the mass of the Earth is  $M_E$ , and their radii are  $r_\odot$  and  $r_E$  respectively, then the ratio of the average densities will be

$$\frac{\rho_E}{\rho_\odot} = \frac{M_E/r_E^3}{M_\odot/r_\odot^3}.$$

(the factor of  $4\pi/3$  for the volume of a sphere doesn't matter because it cancels above and below. The information given is sufficient to work out this ratio. The trick is to tease out of each item of information some combination of parameters relevant to the problem. There are many ways to do this; this is one.

First, let the angular diameter of the Sun be  $\theta = 0.5 \times 2\pi/360 \simeq 0.0087$  radians. Half of this angle is the angle subtended by the radius of the Sun as seen from Earth. Let the Earth's orbital radius (assuming a circular orbit) be  $R$ . Using the small angle approximation gives  $r_\odot/R \simeq \theta/2$ ; so  $R = 2r_\odot/\theta$ .

Next we have that the length of  $1^\circ$  of latitude on the Earth's surface is 100 km. This means that if you move due North in such a way that your latitude changes by  $1^\circ$  then you will have travelled 100 km. This means that if you go all around the Earth you will travel  $360 \times 100$  km, i.e. 36,000 km. This must be  $2\pi r_E$  if the Earth is spherical. Hence  $r_E \simeq 3.6 \times 10^7/2\pi = 6 \times 10^6$  m.

Third there is the length of the year. Using Kepler's Third Law we can relate the period of the Earth's orbit around the Sun to the orbital radius by

$$P^2 = \frac{4\pi^2}{GM_\odot} R^3$$

with  $R$  defined above;  $P = 3 \times 10^7$  seconds. Notice the right hand side involves  $R$  rather than  $r_\odot$ . We can however use the first result to convert it, i.e.

$$\frac{GM_\odot}{R^3} = \frac{4\pi^2}{P^2}$$

means that

$$\frac{GM_\odot}{(2r_\odot/\theta)^3} = \frac{4\pi^2}{P^2}$$

so that

$$\frac{GM_\odot}{r_\odot^3} = \frac{32\pi^2}{\theta^3 P^2} = \frac{32\pi^2}{0.0087^3 \times (3 \times 10^7)^2} \simeq 5.3 \times 10^{-7} \text{s}^{-2}.$$

Finally, we have the acceleration due to gravity at Earth's surface. This must be

$$g = \frac{GM_E}{r_E^2} \simeq 10 \text{m s}^{-2}.$$

We already know  $r_E$  so we can make the ratio

$$\frac{GM_E}{r_E^3} = \frac{10}{r_E} \simeq 1.7 \times 10^{-6} \text{s}^{-2}$$

Notice that both equations involving masses also involve  $G$  so if we take the ratio of  $GM_E/GM_\odot$  the factor of  $G$  will disappear and we don't need to look it up. Hence

$$\frac{\rho_E}{\rho_\odot} = \frac{GM_E/r_E^3}{GM_\odot/r_\odot^3}.$$

which gives

$$\frac{\rho_E}{\rho_\odot} = \frac{1.7 \times 10^{-6}}{5.3 \times 10^{-7}} \simeq 3.3$$

This is only an estimate, so any answers around  $3.5 \pm 0.5$  are acceptable.